Probability

A Measure of Likelihood

While the individual man is an insoluble puzzle, in the aggregate he becomes a mathematical certainty. You can, for example, never foretell what any one man will be up to, but you can say with precision what an average number will be up to. Individuals vary, but percentages remain constant. So says the statistician.
~Arthur Conan Doyle

Probability is a method for turning short term uncertainty into long term prediction.

Can you accurately predict the outcome of these events?

- a coin toss?
- dice roll?
- gender of a fetus?
- height of the next person you see?

Can you predict the long term trend of those events?

Vocab

Experiment: a process that leads to an outcome

Outcome: result of an experiment

Event: a collection of outcomes of an experiment

Sample Space: the set of all possible outcomes

The probability of an event E occurring is given a value between 0 and 1.

$$0 \leq P(E) \leq 1$$

Use proportions where appropriate so you don't lose information.

Round decimals to three decimal places.

Some things to know:

1. If P(E) = 0 then event E will not happen.

2. If P(E) = 1 then event E is certain to happen.

Some things to know:

3. If S is a complete set of events where

$S = \{E_1, E_2, E_3, \ldots E_n\}$

then

$P(E_1) + P(E_2) + P(E_3) + \ldots + P(E_n) = 1$

Which can be written as $\sum_{i=1}^{n} P(E_i) = 1$

Some things to know:

4. If P(not E) is the probability of event E not happening, then

$$P(\text{not } E) = 1 - P(E)$$

P(not E) can also be written $P(\bar{E})$ or $P(\neg E)$

The Main Types of Probability

1. **Classical** *"simple"*

All outcomes are equally likely to occur.

eg in a coin toss heads and tails have equal likelihood of occurring.

P(H) = ½ and P(T) = ½

P(H) + P(T) = 1

The Main Types of Probability

2. **Empirical** *"experimental"*

Relies on experience to determine the likelihood of outcomes.

eg a survey if 7 out of every 10 people interviewed prefer Starbucks to Seattle's Best, what's the probability that the next person interviewed will prefer Seattle's Best?

The Main Types of Probability

3. **Subjective** "*expert opinion*"

Based on opinion or an educated guess.

eg Weather reports, earthquake predictions, sports team performance, stock market.

1. Classical Probability

Given an event E and sample space S

$$P(E) = \frac{\text{total number of ways E can occur}}{\text{total number of elements in S}}$$

eg1 single coin toss

$S = \{H, T\}$

$P(H) = \frac{1}{2}$ $P(T) = \frac{1}{2}$

1. Classical Probability

eg2 two coin toss

S = {HH, HT, TH, TT}

P(HT) = 1/4 P(TT) = 1/4

NB probability cares that HT is different than TH

The standard 6-sided die originated in Egypt and other places about the same time, about 2000 BCE

Gaming "dice" called *astragalus* had been used in the Middle East from about 3600 BCE

1. Classical Probability

Eg 3. 2-child family

S = { BB, BG, GB, GG }

P(BB) = ¼

If the first child is born a boy, is it appropriate to predict the gender of the next child?

eg4 The candy jar contains a mixture of Jolly Rancher Candies. There are five red, two yellow, and three green. If you pick a candy without looking find these probabilities.

P(it is red) =

P(it is green) =

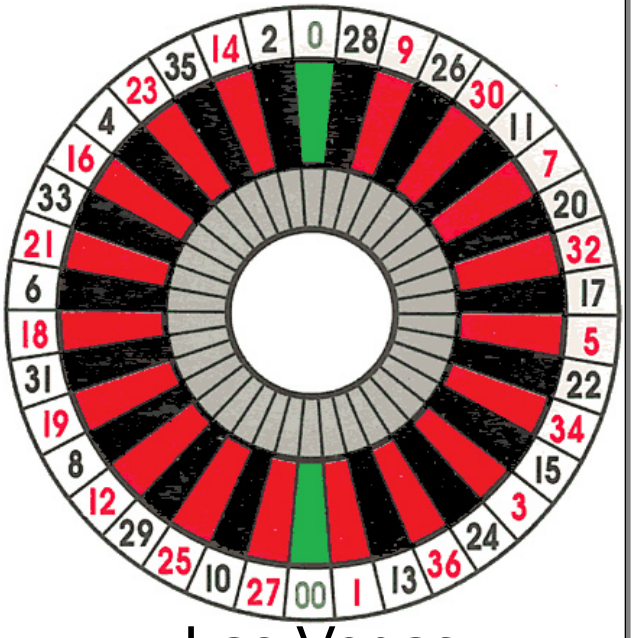
P(if is red or green) =

P(it is not green) =

P(it is not red) =

The roulette wheel originated in France about 1800 CE.

Europe



Las Vegas

Russian

eg5 A roulette wheel in a casino has 38 spaces numbered 1 through 36, 0, and 00. In a spin of the roulette wheel a ball will eventually land on one of the 38 spaces. There are many bets you place, including guessing which number the ball will land on, though you cannot bet on the 0 or 00.

In a spin of the wheel, find these probabilities.

P(An odd number)

P(A number greater than 25)

P(A number less than 15, not counting 0 or 00)

eg6 In a family with 4 children, what is:

P(2 girls)

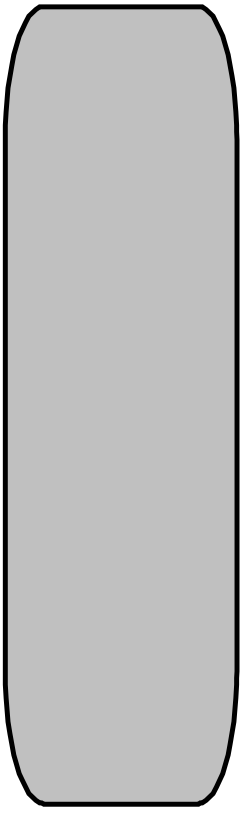
P(4 girls)

P(No girls)

P(All of one gender)

P(2 girls and 2 boys)

P(3 of one gender)

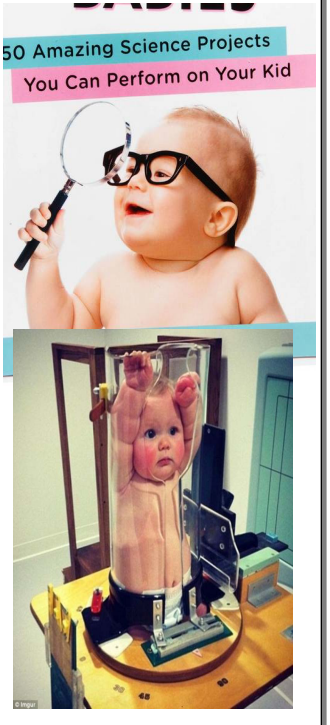


A woman and a man (unrelated) each have two children. At least one of the woman's children is a boy, and the man's older child is a boy.

Do the chances that the woman has two boys equal the chances that the man has two boys?

2. Empirical Probability

•The likelihood or probability of outcomes is determined by experience or experiment.



For example
•Surveys
•Lab experiments
•Observational studies, usually long term

Given event E

$$P(E) = \frac{\text{frequency of E occurring}}{\text{number of trials conducted}}$$

A random sample of 100 LHS students were asked the question:

"What mode of transport did you use to get to school today?"

It was found that 57 came by car, 28 came by mass transit (MT), 12 walked, and 3 came by "other".

If a student is chosen at random find:

P(they came by MT) =

P(came by car) =

P(came by car and MT) =

P(came by car or MT) =

P(came by not walking) =

Who prefers Pepsi products to Coke products?

If a person from this class is chosen at random find these probabilities:

P(the person prefers Pepsi products)

P(the person prefers Coke products)

P(the person does not prefer Coke products)

3. Subjective Probability

Expert opinion
Educated guess

eg1 There is a 50% chance of rain.

eg2

"The ability to anticipate the supply and demand of currency has a success rate no better than that of forecasting the outcome of a coin toss."

Allan Greenspan
ex-Chairman of the Federal Reserve
11/19/2004

March 13, 2018

In an office there are five women and four men. If a person is selected at random find these probabilities:

P(woman) =

P(man) =

P(man or woman) =

P(man and woman) =

In a cooler there are 12 cans of cola and 6 cans of root beer. If you pick a can at random what is the probability that it is cola?

In a different cooler there are 23 cans of cola and an unknown number of cans of ginger ale. If the probability of randomly picking a can of ginger ale is 0.4524 how many cans of ginger ale are there in the cooler?

A single card is drawn from a standard deck of 52 cards.
Find these probabilities.

P(♠) =

P(6) =

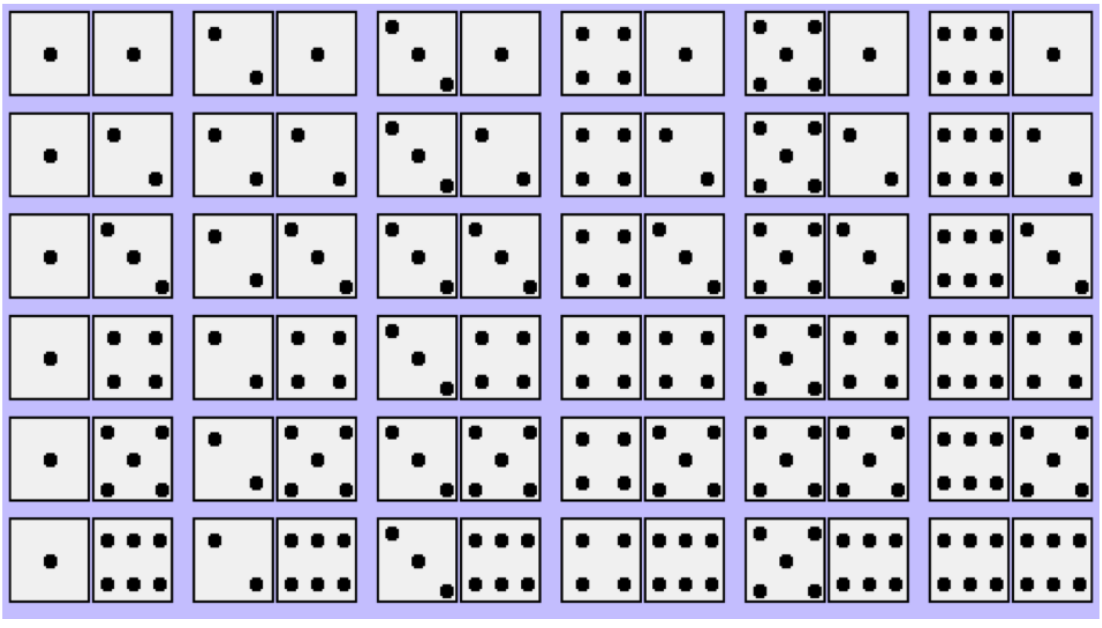
P(Q♦) =

P(♥ or ♣) =

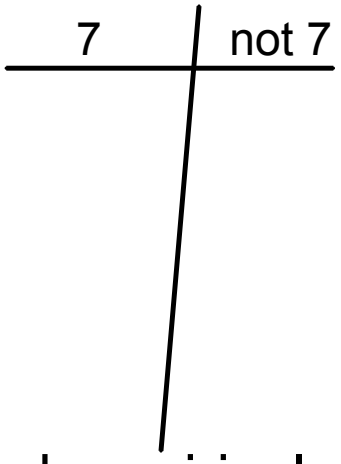
P(K or Q) =

P(A♠) =

36 Possible Results with Two Dice



What is the P(sum of 7)?



Let's see if it's true.

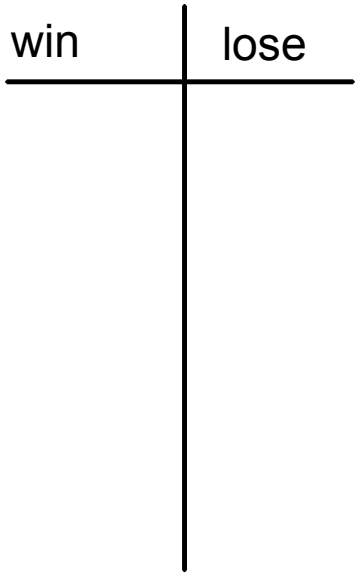
Doing a calculator simulation to find empirical probability.

Each of you need to do this so we can amass enough results to be useful.

In the game of craps, using two dice, a person wins on the first roll if a 7 or an 11 is rolled.

Find the empirical probability of winning.

P(7 or 11) =



In the game of craps, using two dice, a person wins on the first roll if a 7 or an 11 is rolled.

Find the classical P(win on 1st roll) =

Does this match the empirical probability?

In the game of craps a person loses on the first roll if a 2, 3, or 12 is rolled.

Find the empirical probability of losing on the first roll.

Compare this to the classical probability.

P(2 or 3 or 12) = 4 / 36

There is 1 way to roll a 2,
2 ways to roll a 3, and
1 way to roll a 12.

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Probability Simulation exercise

Chevalier de Méré

aka

Antoine Gombaud

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